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Phytoplankton-nutrients interaction in Ypacarai shallow lake

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1 Introduction

The Ypacarai lake is one of the most important water resources in Paraguay. At the moment, its physicochemical and biological characteristics show that its aquatic ecosystems are degraded in eutrophic state. Furthermore, the large amount of effluents discharged into the lake is the main cause of its pollution. Effluents carry nutrients in the form of nitrogen and phosphorus, which promote phytoplankton growth. In this work we model the kinetics of phytoplankton growth and its interaction with nitrogen and phosphorus, in order to predict zones with high eutrophication.

2 Modeling

The hydrodynamics of the lake is modeled through the 2D shallow water equations, where equation (1) is the continuity equation and equation (2) is the momentum equation. We include the wind and bottom stresses only and neglect the Coriolis and turbulence terms. Equation (3) shows the depth averaged scalar transport equation, to model total phosphorus, total nitrogen and phytoplankton.

$$\frac{\partial h}{\partial t} + \frac{\partial (u_i h)}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -g \frac{\partial}{\partial x_i}(h + z_0) + \frac{1}{h\rho} \frac{\partial}{\partial x_j}(h\tau_{ij}) - \frac{1}{h} \frac{\tau_{so,i}}{\rho} + \frac{1}{h} \frac{\tau_{wind,i}}{\rho} \tag{2}$$

$$\frac{\partial (hc_i)}{\partial t} + u_i \frac{\partial (hc_i)}{\partial x_j} = D \frac{\partial^2}{\partial x_j^2}(hc_i) + g_i + f_i \tag{3}$$

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where $u_i : i \in \{1, 2\}$ are the depth-averaged flow velocities (u, v) in x and y, h is the water depth, z_0 is the bed elevation, τ is the momentum diffusion term, τ_{so} is the horizontal component of the bed friction, τ_{wind} is the component of the wind stress, ρ is the water density and g the gravity acceleration. Finally, $c_i : i \in \{1, 2, 3\}$ are the scalar concentrations of nitrogen, phosphorus and phytoplankton, D is the diffusion coefficient, g_i is the term modeling the consumption rates of phosphorus and nitrogen, and production rate of phytoplankton, and f_i includes other terms such as phosphorus sedimentation and inlets/outlets.

3 Conclusion

Simulated values of nutrients and phytoplankton distributions are contrasted with government measurements showing similar results. This analysis enables the detection of high polluted zones based on the complete distribution of phytoplankton in the lake. The results are useful in the definition of environmental management actions focusing on reducing damage to the aquatic and terrestrial ecosystem.

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References

[1] Józsa J. Shallow lake hydrodynamics. Theory, measurement and numerical model applications. Lecture Notes of the IAHR short course on environmental fluid mechanics, Budapest, pages 7-16, 2004.

[2] Fragoso, C. R., Marques, D. M. M., Collischonn, W., Tucci, C. E., van Nes, E. H.. Modelling spatial heterogeneity of phytoplankton in Lake Mangueira, a large shallow subtropical lake in South Brazil. *Ecological modelling, 219(1), 125-137*, 2008. DOI:10.1016/j.ecolmodel.2008.08.004

[3] L. Oporto, C. E. Schaerer, Wind effects on contaminant transport in shallow lakes, *Proceeding Series of the XXXVI Iberian Latin-American Congress on Computational Methods in Engineering*, 2015.

[4] Oporto L., Ramírez D., Varela J., Schaerer C. Analysis of contaminant transport under wind conditions on the surface of a shallow lake. *Mecánica Computacional*, volume XXXIV , 2155-2163, 2016.

[5] Tsanis I.K., Wu J., Shen H., and Valeo C. Environmental hydraulics: hydrodynamic and pollutant transport modelling of lakes and coastal waters. Developments in Water Science, 56:vii-360, 2006.